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(under the assumption that $\varphi(t)$ is continuous at $t = 0$), or what is the same

$$\lim_{n=\infty} n\int_0^1 (1-t)^{n-1}[\varphi(t)-\varphi(0)]dt = 0.$$

Let $\epsilon > 0$ be as small as we please; $\varphi(t)$ being continuous at $t = 0$, it is then possible to choose a δ, $0 < \delta < 1$, and so small that $|\varphi(t) - \varphi(0)| < \epsilon/2$ for $0 \leqq t \leqq \delta$. Let M be greater than $|\varphi(t) - \varphi(0)|$ for $0 \leqq t \leqq 1$. Then, decomposing the integral into two parts and using the first mean value theorem,

$$\left| n\int_0^1 (1-t)^{n-1}[\varphi(t)-\varphi(0)]dt \right| \leqq \left| n\int_0^\delta \right| + \left| n\int_\delta^1 \right|$$

$$\leqq n\frac{\epsilon}{2}\int_0^\delta (1-t)^{n-1}dt + nM\int_\delta^1 (1-t)^{n-1}dt$$

$$< \frac{\epsilon}{2} + M(1-\delta)^n.$$

Now N may be chosen so large that $M(1-\delta)^n < \epsilon/2$ for $n > N$, so that finally

$$\left| n\int_0^1 (1-t)^{n-1}[\varphi(t)-\varphi(0)]dt \right| < \epsilon \text{ for } n > N,$$

which completes our proof.

192 (Number Theory) [1913, 196; 1919, 214]. Proposed by the late ARTEMAS MARTIN.

Find rational values of v, w, and x that will satisfy simultaneously the conditions

$$(m^2+n^2)(v^2+w^2+x^2)^2 - 4m^2n^2v^2 + m^2n^2(m^2+n^2) = 0,$$

$$(m^2+n^2)(v^2+w^2+x^2)^2 - 4m^2n^2w^2 + m^2n^2(m^2+n^2) = 0,$$

$$(m^2+n^2)(v^2+w^2+x^2)^2 - 4m^2n^2x^2 + m^2n^2(m^2+n^2) = 0,$$

m and n being known rational quantities.

DISCUSSION BY H. S. UHLER, Yale University.

The following analysis shows that the problem is impossible.

Any set of values of v, w, and x that fulfil the given conditions must also satisfy the sum of the three equations, that is, must satisfy

$$3(m^2+n^2)(v^2+w^2+x^2)^2 - 4m^2n^2(v^2+w^2+x^2) + 3m^2n^2(m^2+n^2) = 0.$$

From this equation, we have

$$v^2+w^2+x^2 = \frac{mn}{3(m^2+n^2)}[2mn \pm i\sqrt{9m^4+14m^2n^2+9n^4}].$$

In general, the right hand member is irrational, and the sum of the squares of any number of rational numbers cannot have an irrational value; therefore, the problem is impossible.

We may proceed farther and find explicit expressions for v, w, and x. By inspection, or by forming the differences between the given conditions, taken in pairs, we see that $v^2 = w^2 = x^2$. Consequently,

$$x^2 = \frac{mn}{9(m^2+n^2)}[2mn \pm i\sqrt{9m^4+14m^2n^2+9n^4}],$$

or

$$x = \pm\frac{1}{3}\sqrt{\frac{mn}{2(m^2+n^2)}}\,\{\sqrt{3(m^2+n^2)+2mn} \pm i\sqrt{3(m-n)^2+4mn}\}.$$

Obviously, these four roots are irrational, in general.